

Study of Flavor Diagonal CP Violations on Lattice

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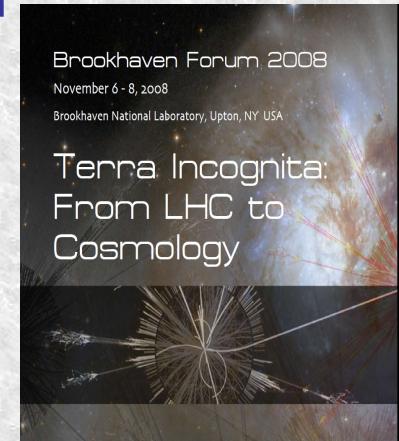
- R. Zhou, T. Blum, T. Doi, M. Hayakawa, Tl, N. Yamada
(Riken-BNL-Columbia collab.) arXiv:0810.1302, PRD76:114508.
- S. Aoki, R. Horsley, Tl, Y. Nakamura, D. Pleiter, P. Rakow,
G. Schierholz, J. Zanotti arXiv:0808.1428, arXiv:0802.1470

θ θ θ

New Physics search on LHC and beyond

- Baryon dominance in the universe requires extra CP-violation (Sakharov criteria)
- Success of Cabibbo-Kobayashi-Maskawa theory, flavored CP violation is well described so far.
- Fine tuning in the flavor diagonal CP violation
 - Dimension 4: (the Strong CP problem)
 $\theta F^{\text{QCD}} \tilde{F}^{\text{QCD}} = \theta F_{\mu\nu}^{\text{QCD}} F_{\rho\lambda}^{\text{QCD}} \epsilon_{\mu\nu\rho\lambda}$
 - Dimension 5: (can't be rotated away by axion)

$$\sum_{i=u,d,s,e,\mu} d_i^{\text{QED}} \bar{\psi}_i F_{\mu\nu}^{\text{QED}} \sigma_{\mu\nu} \gamma_5 \psi_i + \sum_{q=u,d,s} d_q^{\text{QCD}} \bar{q} F_{\mu\nu}^{\text{QCD}} \sigma_{\mu\nu} \gamma_5 q$$



EDM Experiments

- Neutron

$$|d_N| < 2.9 \times 10^{-26} \text{ e-cm}$$

Baker et al. (2007)

- ^{199}Hg

$$|d_{\text{Hg}}| < 2 \times 10^{-28} \text{ e-cm}$$

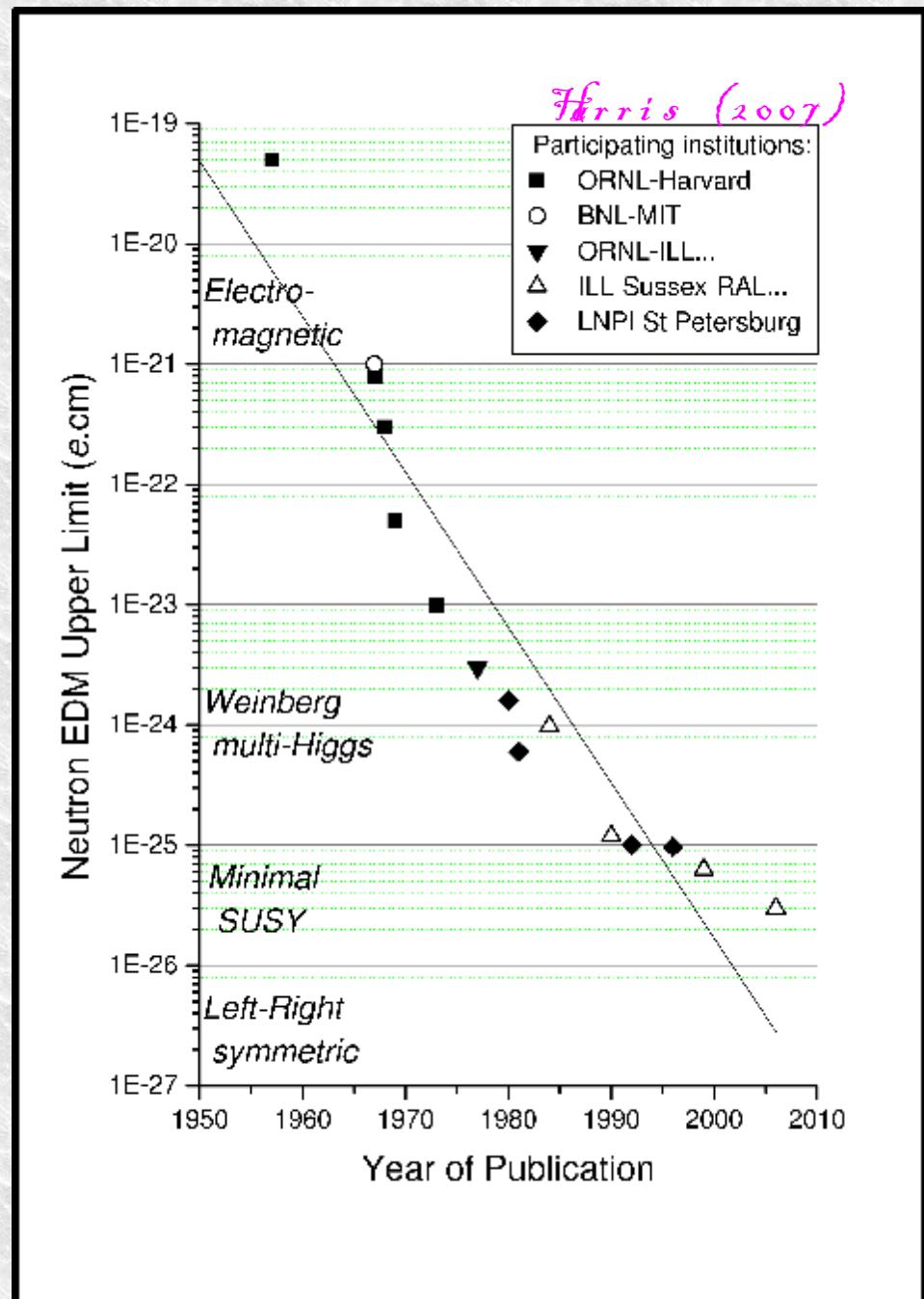
Romalis et al. (2000)

- ^{205}Tl

$$|d_{\text{Tl}}| < 9 \times 10^{-25} \text{ e-cm}$$

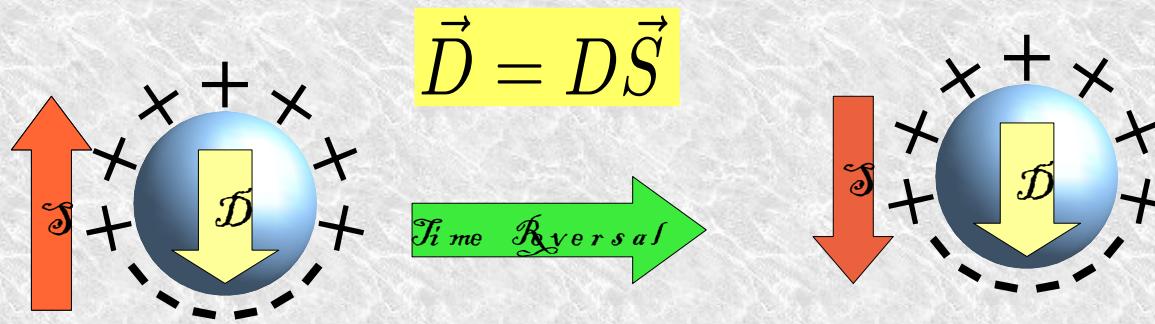
Regan et al. (2002)

- Many other plans including BNL Storage Ring EDM collaboration deuteron EDM $\sim 10^{-29}$ e-cm



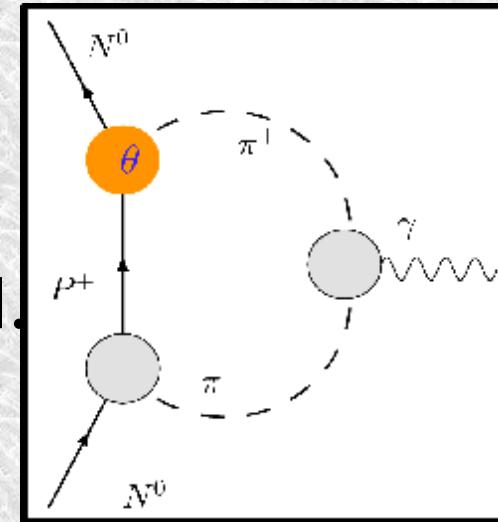
Electric Dipole Moment (EDM)

- Permanent EDM is a signature of CP (or Time reversal) symmetry violation.



- various candidates of CP violations :
 - Electro Weak (CKM phase in quark mass matrix) : very small (Neutron: 10^{-4} smaller than exp. limit)
 - New Physics (axion, SUSY, left-right, multi Higgs)
 - Strong CP: vacuume angle θ This talk

Strong CP problem



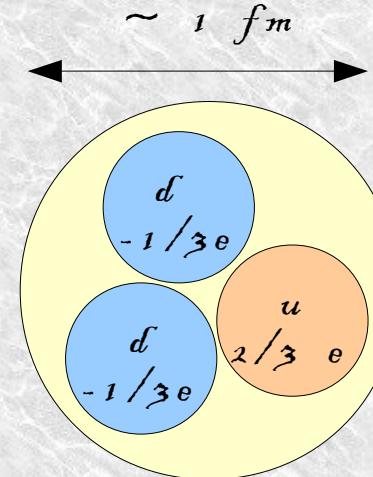
- CP odd Lagrangians in QCD are perfectly allowed.

$$S_\theta = i\theta \frac{1}{32\pi^2} \int d^4x F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) = i\theta Q_{\text{top}}$$

Q_{top} is (classically) integer counting winding number of gluon field (net number of instanton). θ is an angle of complex phase assoc. each topological sectors: θ vacuum

- Neutron's Electric Dipole Moment, NEDM (CP odd) is experimentally very small,

$$D_n \leq 2.9 \times 10^{-13} [\text{e fm}] \quad (\text{07, Grenoble grp.})$$



- Model estimates (Bag model, ChPT, Large N_c)

$$D_n = (9, 17, -4 \sim -7) \times 10^{-3} \times \theta [\text{e fm}]$$

(79 Baluni, 79 Crewther et.al., 92 Aoki-Hatsuda, 02 Pospelov-Ritz)
naturalness

$$|\bar{\theta}| = |\theta + \theta_{\text{EW}}| \leq 10^{-10}$$

Proposed solutions

- theta term is U(1) ABJ Anomaly:
changes the path integral measure

$$\begin{aligned}\psi &\rightarrow e^{i\theta' \gamma_5} \psi, \\ \bar{\psi} &\rightarrow \bar{\psi} e^{i\theta' \gamma_5}\end{aligned}$$

$$\mathcal{D}\psi \mathcal{D}\bar{\psi} \rightarrow \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i \frac{N_f}{32\pi^2} \theta' F_{\mu\nu} \tilde{F}_{\mu\nu}}$$

and also quark mass terms

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}\psi \cos(2\theta') + im\bar{\psi}\gamma_5\psi \sin(2\theta')$$

- if up(/down) quark is massless ($m=0$) then vacuum angle can be rotated away, which is less consistent with spectrum results as we will see in the next pages. (Creutz Ann. Phys. 322:1518, 2007)
- A very light, neutral, weakly coupled pseudoscalar, axion, which is (would-be) NG-boson of the Peccei-Quinn Symmetry

$$\left(\theta - \frac{a(x)}{f_A} \right) F_{\mu\nu} \tilde{F}_{\mu\nu}$$

QCD+QED simulation

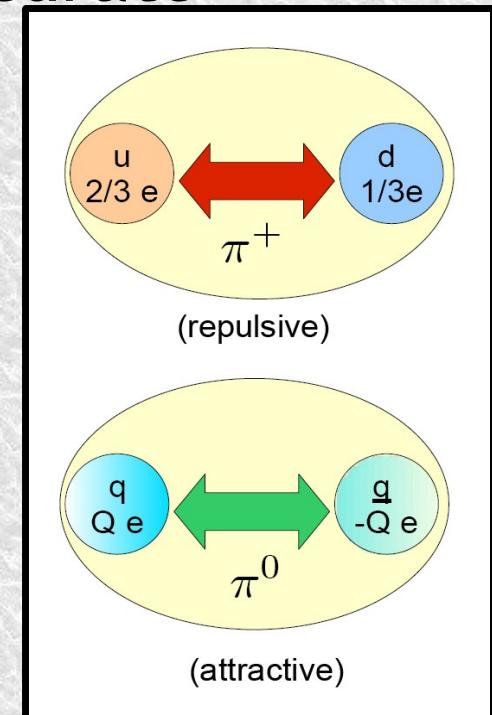
- The first principle calculations of **isospin breaking** effects due to **electromagnetic(EM)** and the up, down quark mass difference are necessary for accurate determination of quark masses.

- EM splittings are measured very accurately:

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5) \text{ MeV}$$

$$m_N - m_P = 1.2933317(5) \text{ MeV}$$

- $O(\alpha)$ radiative corrections in Hadron structure is **a major uncertainties**
e.g. $f_{\pi^\pm} = 130.7 \pm 0.1 \pm 0.36 \text{ MeV}$ PDG



EM splittings

- ChPT with EM at $O(p^4, p^2 e^2)$:

Urech, NPB433 (95) 234 Bijnens et al. PRD (07) 014505

$$\begin{aligned} M_{\pi^\pm}^2 &= 2mB_0 + \cancel{2e^2} \frac{C}{f_0^2} + \mathcal{O}(m^2 \log m, m^2) + I_0 e^2 m \log m + K_0 e^2 m \\ M_{\pi^0}^2 &= 2mB_0 + \mathcal{O}(m^2 \log m, m^2) + I_\pm e^2 m \log m + K_\pm e^2 m \end{aligned}$$

- Neutral pion, kaons are still **Nambu-Goldstone boson** @ $m_q = 0$
- Dashen's theorem**: The difference of squared pion mass is independent of quark mass upto $O(e^2 m)$

$$\Delta M_\pi^2 \equiv M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 \frac{C}{f_0^2} + (I_\pm - I_0)e^2 m \log m + (K_\pm - K_0)e^2 m$$

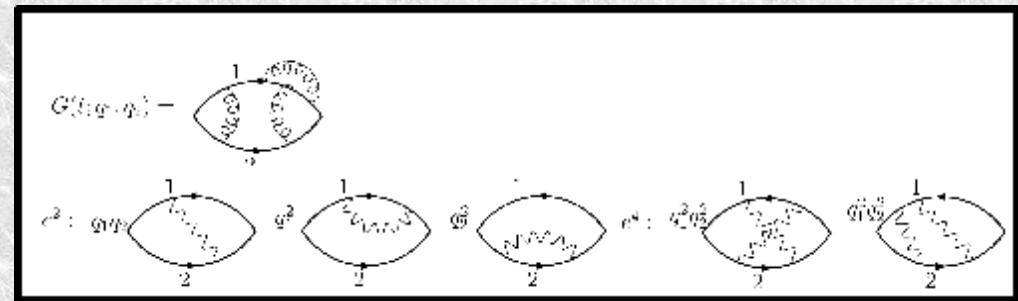
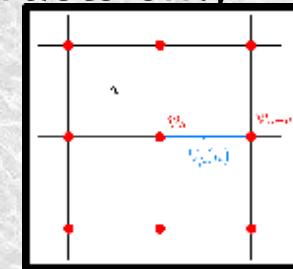
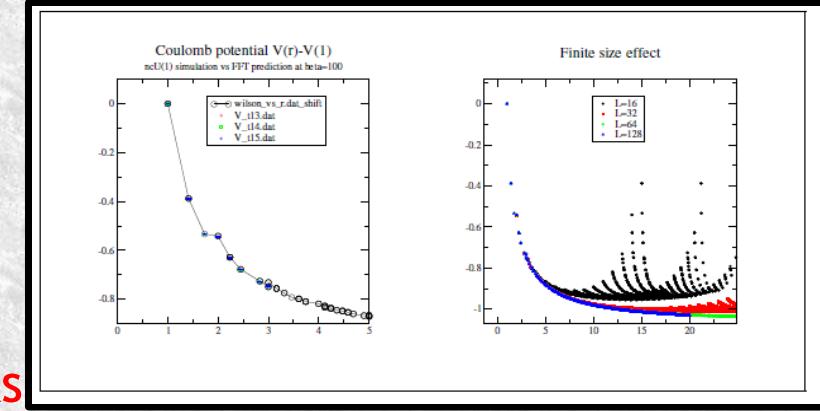
- C, K_\pm, K_0 are new low energy constants determined on **QED+QCD simulation** (I_\pm, I_0 are known functions).

EM splittings on lattice

- Full QCD: Nf=2 (& 2+1) dynamical DWF quarks
- Free photon field (non-compact lattice U(1) gauge field) interacts only with valence quark (dynamical QCD but quenched QED)
- Lattice spacings $a^{-1} = 1.69(53)$ [GeV] or $a \sim 0.12$ [fm]
Volume $\sim (1.9 \text{ fm})^3$
- About 200 x 2 statistical samples of QCD x QED vacuum for each quark mass points (50, 75, 100 % of strange quark mass) are used to get the Hadron propagators .

$$C_X(t) = A(e^2) e^{-M_X(e^2)t}$$

$$\frac{C_{X^\pm}(t) - C_{X^0}(t)}{C_{X^0}(t)} = \Delta M_X \times t + \text{Const.}$$

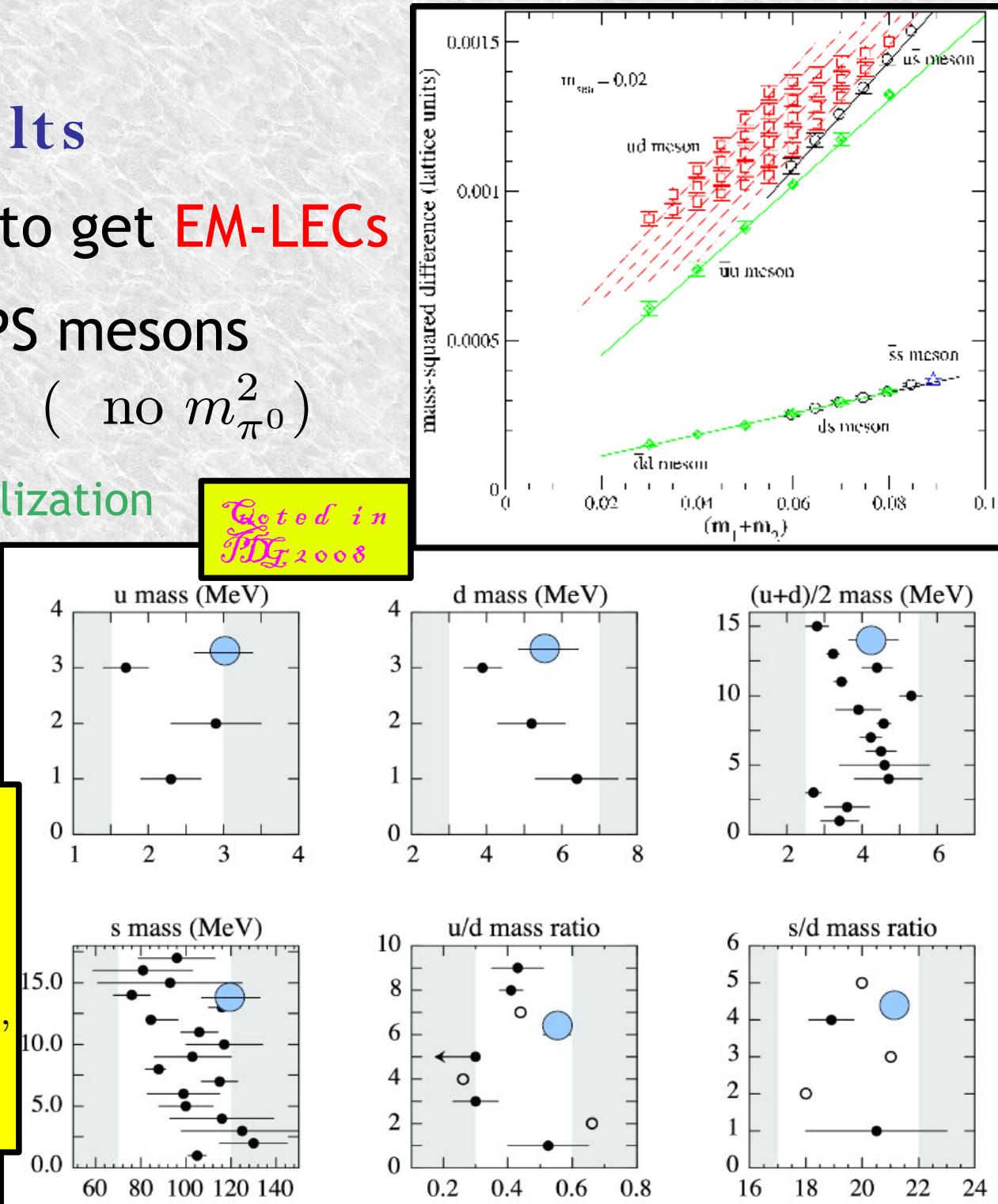


EM splittings results

- Fit 61×3 PS masses to get EM-LECs
- Input experimental PS mesons
 $m_{\pi^\pm}^2, m_{K^\pm}^2, m_{K^0}^2$ (no $m_{\pi^0}^2$)
- Non-perturbative renormalization
- Massless quarks are unlikely consistent with experiments

$m_u^{\overline{\text{MS}}}(2 \text{ GeV}) = 3.02(27)(19) \text{ MeV},$
 $m_d^{\overline{\text{MS}}}(2 \text{ GeV}) = 5.49(20)(34) \text{ MeV},$
 $m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) = 4.25(23)(26) \text{ MeV},$
 $m_s^{\overline{\text{MS}}}(2 \text{ GeV}) = 119.5(56)(74) \text{ MeV},$
 $m_u/m_d = 0.550(31),$
 $m_s/m_{ud} = 28.10(38).$

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Nucleon EDM on Lattice

- 18 years of Nucleon Electric Dipole Moment,
NEDM on lattice : two methods so far

	$\mathcal{N}=0$	$\mathcal{N}=0$	Electric field
09 Akti-Gocksch	Wils on		
04 Ferruto et al.	DWF		3pt
05 Ferruto et al.	$\mathcal{N}=2$ DWF		3pt
06 Shintani et al. (CP-PACS)	$\mathcal{N}=0$ DWF		3pt
06 Shintani et al. (CP-PACS)	$\mathcal{N}=0$ DWF, $\mathcal{N}=2$ clover	Electric field	
06 QCDSF	$\mathcal{N}=0$ overlap		3pt
06 Bum, JJ & Doi	$\mathcal{N}=2$ DWF		Electric field
06 Shintani et al. (CP-PACS)	$\mathcal{N}=2$ clover		Electric field

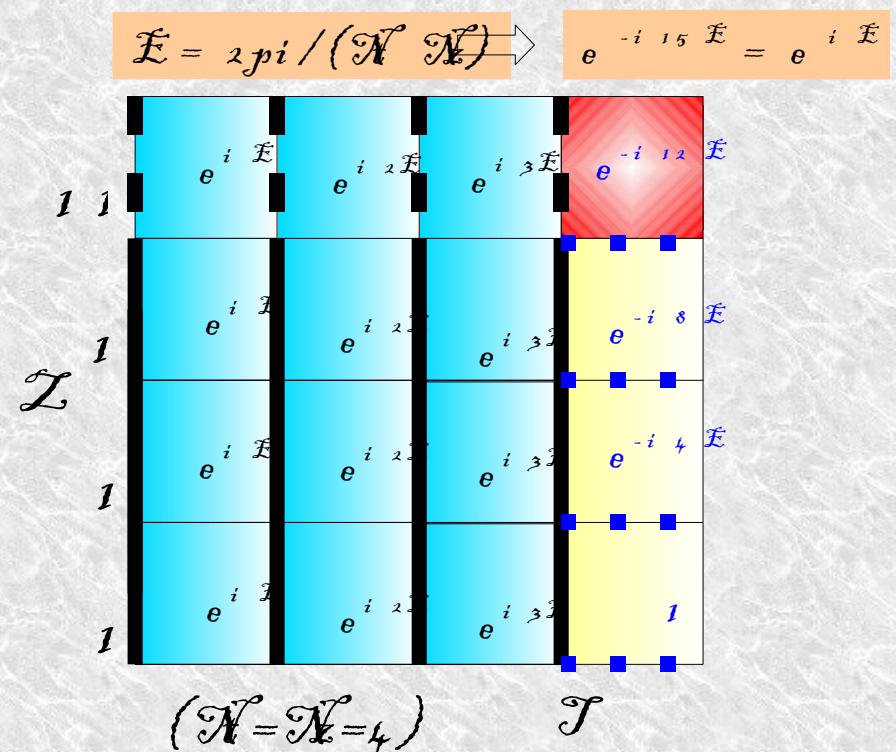
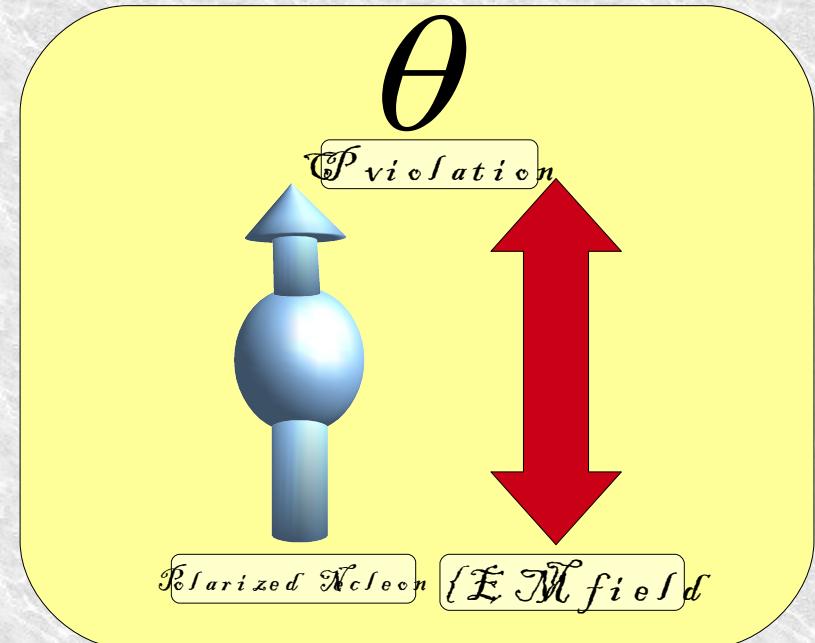
(c.f. experiments: 10^{-6} progress for last ~50 years)

- Dynamical Simulation (including sea quark pair creation/annihilation effect) becomes easier. NEDM is very sensitive to **sea quark mass**; zero at chiral limit (and valence quark theta is to be rotated away)

NEDM simulation

- Three ingredients
 - Uniform Electric Field by a boundary condition
(98 U. Heller, 64 E. Brown)
 - source of CP violation θ
 - Polarized Nucleon
- Measure a spin splitting of Nucleon's energy
(99 Aoki, Cocksch)

$$m_{N^\theta}(E, \uparrow) - m_{N^\theta}(E, \downarrow) = 2i\theta d_N \mathbf{S} \cdot \mathbf{E} + \mathcal{O}(E^2, \theta^2)$$



Dynamical QCD with θ

- Source of CP violation : $S_\theta = i \frac{\theta}{32\pi^2} \sum \tilde{F}F = i\theta Q_{\text{top}}$
- In previous calculations, $\theta = 0$ ensemble were generated ($N_f=0,2$), then each configuration were reweighted by topological charge : $\langle \mathcal{O} \rangle_\theta = \langle \mathcal{O} e^{i\theta Q} \rangle_0$
- Alternatively, one could put θ term into the ensemble probability, rather than observable, if θ is analytically continued to pure imaginary.
- Spin splitting becomes real:

$$m_{N^\theta}(E, \uparrow) - m_{N^\theta}(E, \downarrow)$$

$$= 2i\theta d_N S \cdot E \rightarrow -2\theta d_N S \cdot E$$

previously done by Minkowskian E .

$$\theta \rightarrow -i\theta$$

- 85 Bhanot-David, 02 Azcoiti et. al,
06 Imachi-Yoneyama et.al.
(CP^N model with imaginary theta)
- 02 de Forcrand-Philipsen,
03 D'Elia-Lombardo (chem. pot.)

Simulation with imaginary θ term

- By using anomalous chiral WT theta term is absorbed into CP-odd “quark mass term”.

$$\begin{aligned}\bar{\psi} &\longrightarrow \bar{\psi} e^{\theta \gamma_5} \\ \psi &\longrightarrow e^{\theta \gamma_5} \psi\end{aligned}$$

$$\mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S_\theta} \rightarrow \mathcal{D}\psi \mathcal{D}\bar{\psi}$$

(flavor-singlet axial “ \mathcal{U}_1 ”) rotation)

$$\mathcal{L}_\theta = \frac{m\theta}{2} \bar{\psi} \gamma_5 \psi$$

- Nf=2 clover fermion

$$\bar{\psi} [D_W + m + i a \frac{c_{SW}}{4} \sigma_{\mu\nu} F_{\mu\nu}] \psi(x)$$

- $V \sim (2 \text{ fm})^3 (16^3 \times 32)$

- $a^{-1} = 2 \text{ GeV}$ (beta=2.1 Iwasaki)

$$F_{\mu\nu}^C = \frac{1}{4} \text{An} \left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \right), \quad F_{\mu\nu}^R = \frac{1}{16} \text{An} \left(\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \end{array} \right),$$

- Quark mass: $mPS/mV = 0.85$

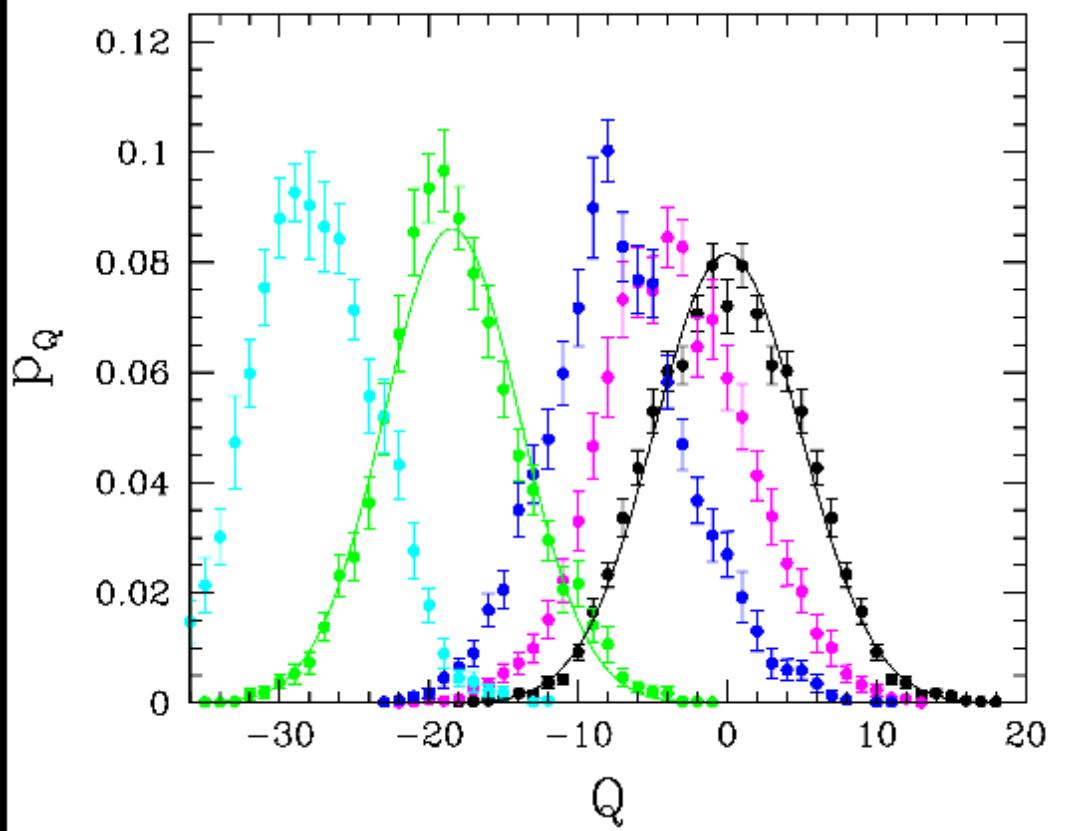
- $\theta = 0.4, 0.0, (0.2)$

- $\sim 5,000/50$ vacuum samples for each parameter set

$$\theta'' = \left[1 - \frac{\kappa}{\kappa_c(\beta)} \right] Z_P \theta$$

Topological charge distribution

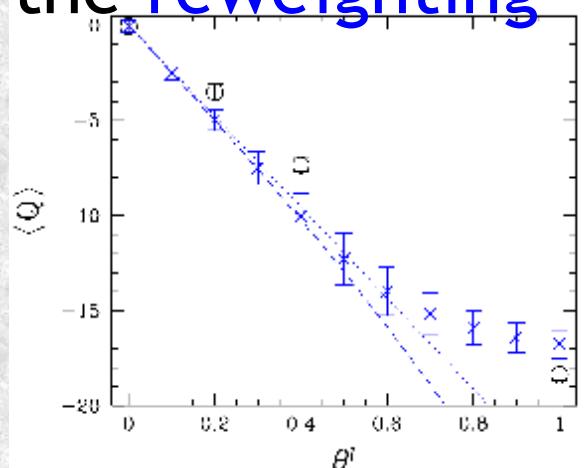
$\theta = 1.5, 1.0, 0.4, 0.2, 0.0$



$$Q_{\text{top}} = \frac{1}{32\pi^2} \sum \tilde{F} F(x)$$

$$F_{\mu\nu}^C = \frac{1}{4} \text{An} \left(\begin{array}{|c|c|} \hline & \square \\ \hline \square & \square \\ \hline \end{array} \right), \quad F_{\mu\nu}^R = \frac{1}{16} \text{An} \left(\begin{array}{|c|c|c|c|} \hline & \square & \square & \square \\ \hline \square & & & \\ \hline & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \square & \square & \square \\ \hline \square & & & \\ \hline & & & \\ \hline \end{array} \right),$$

- Topological charge Q_{top} from gluon field (APE cooling)
- CP symmetry is successfully **broken** by the imaginary θ .
- Roughly consistent with the **reweighting** data



Preliminary results of NEDM

- The third smallest Uniform Electric field

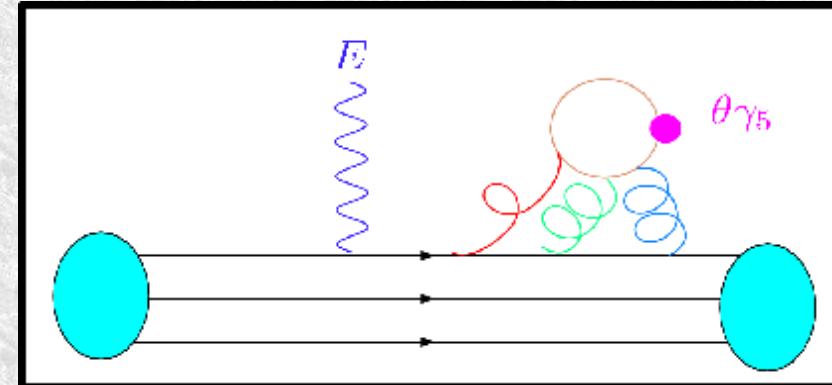
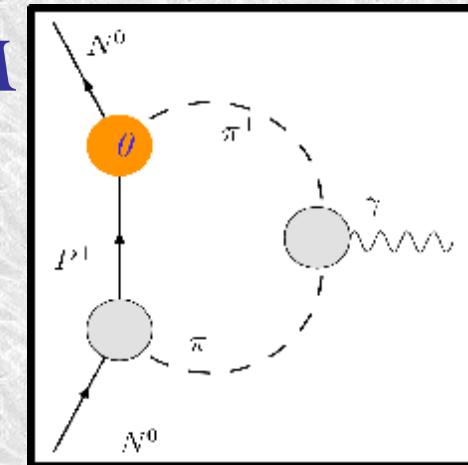
$$E_Z = \frac{2\pi}{N_z N_t} \times 3 \simeq 0.0368$$

- quark EM charges = {+,-} 1, 2 \times {2/3, -1/3}
- tsrc = 0, 16, Nucleon 2pt corr. func. (4x4)

$$\frac{\langle N_\uparrow(t) \bar{N}_\uparrow(t_{src}) \rangle}{\langle N_\downarrow(t) \bar{N}_\downarrow(t_{src}) \rangle} \rightarrow c e^{\Delta m t}$$

$$\Delta m = m_{N^\theta}(E, \uparrow) - m_{N^\theta}(E, \downarrow)$$

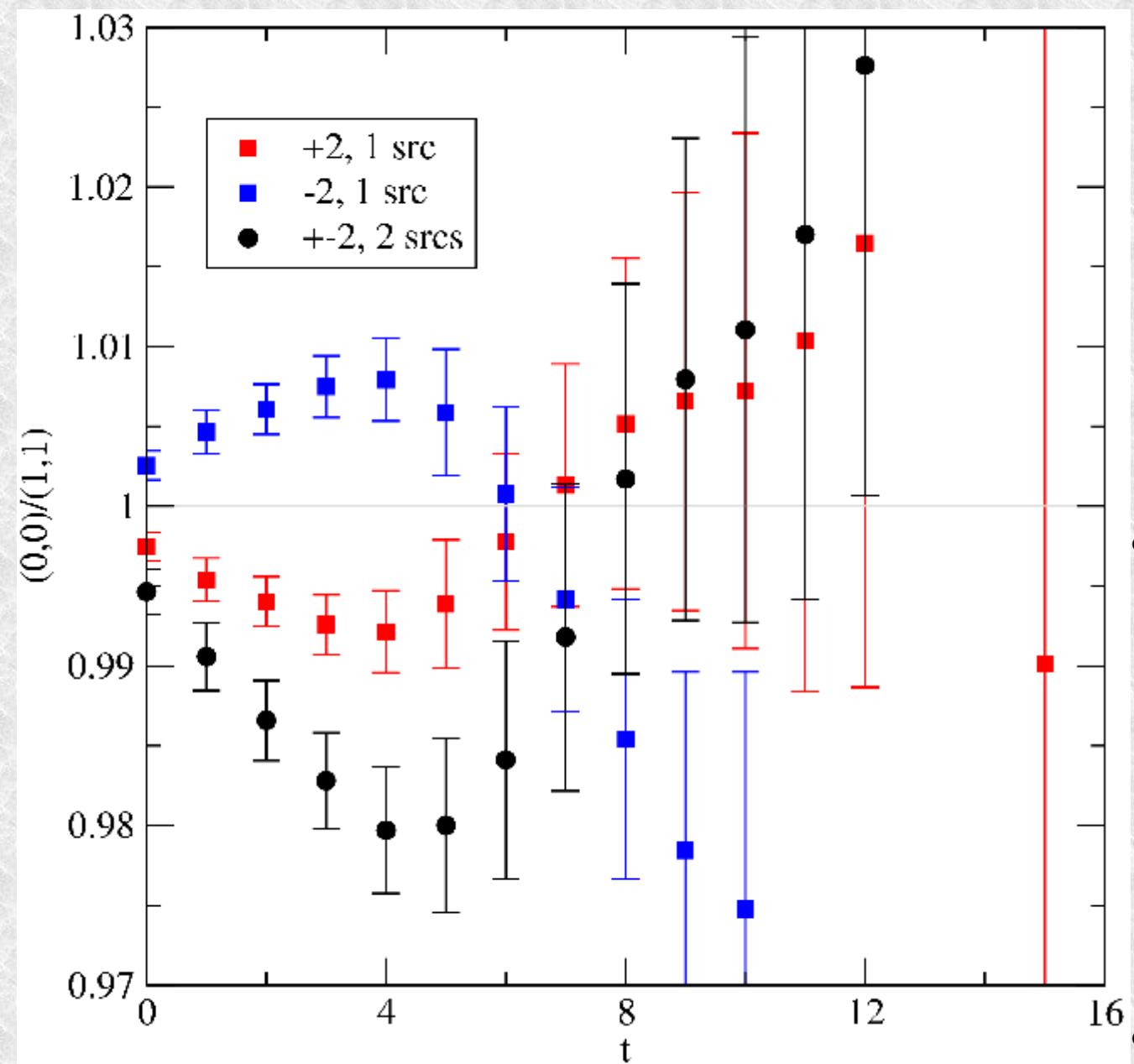
$$= -2\theta d_N \mathbf{S} \cdot \mathbf{E}$$



- Spin splittings of the Nucleon propagator

$$\frac{\langle N_\uparrow(t) \bar{N}_\uparrow(t_{src}) \rangle}{\langle N_\downarrow(t) \bar{N}_\downarrow(t_{src}) \rangle} \rightarrow c \exp(\Delta m t) \simeq c(1 + \Delta m t)$$

- Signal gain by
 - taking ratio of the propagators
 - averaging +/- E, cancels $O(E^0)$
 - many sources
- signal at $t < 5$ excited state contamination needs to be checked

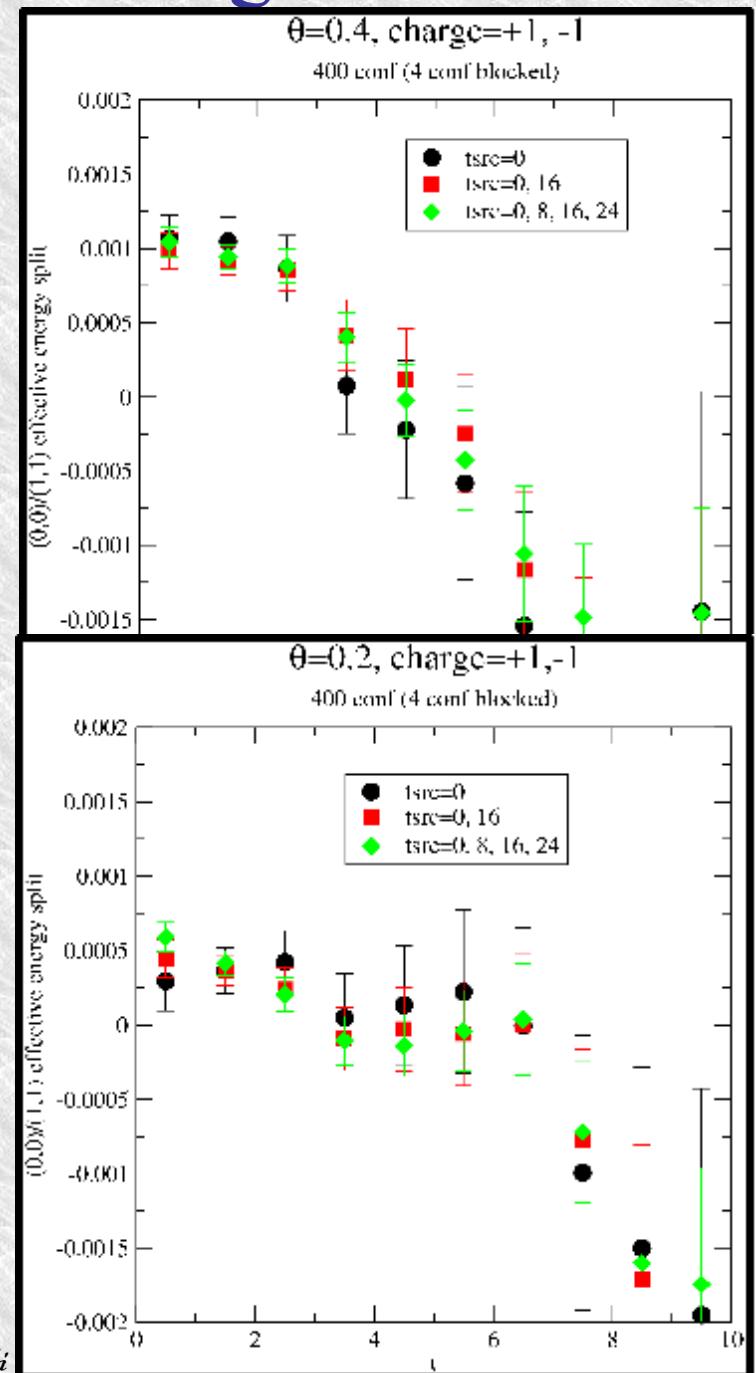


effective energy splitting

$$\Delta m = m_{N^\theta}(E, \uparrow) - m_{N^\theta}(E, \downarrow)$$

$$= -2\theta d_N \mathbf{S} \cdot \mathbf{E}$$

- average t, Nt-t at propagator level
- numbers are comparable to the previous estimations
- scaling with theta indicates physical signal (no correlation)

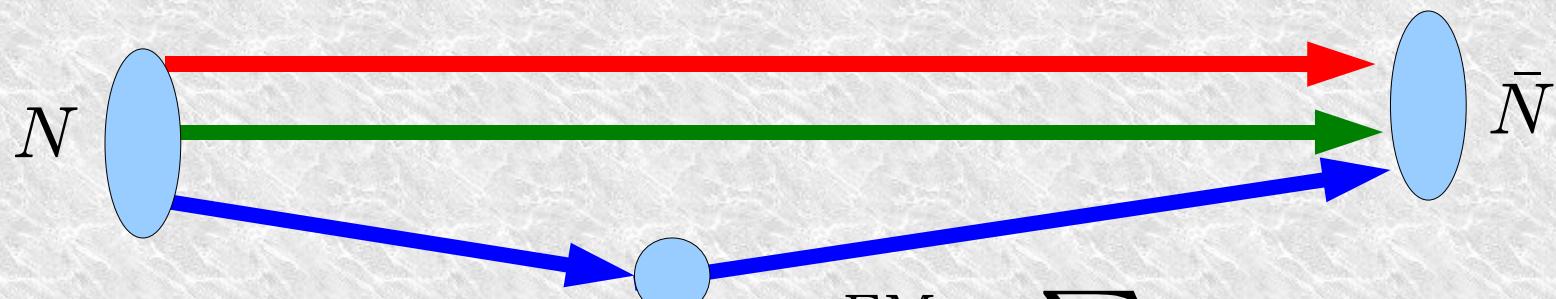


EDM from form factors (3 pt function)

- Electric form factors F3

(04 Berruto, Blum, Orginos, Soni, 05 Shintani et.al CP-PACS)

$$\begin{aligned} \left\langle N_s(p') | V_\mu^{EM}(q) | \bar{N}_s(p) \right\rangle_\theta &= F_1(q^2) \gamma_\mu + F_2(q^2) \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \\ &+ i\theta F_3(q^2) \frac{q_\nu \sigma_{\mu\nu} \gamma_5}{2m_N} + \dots, \quad q = p' - p \end{aligned}$$



- EDM is obtained by

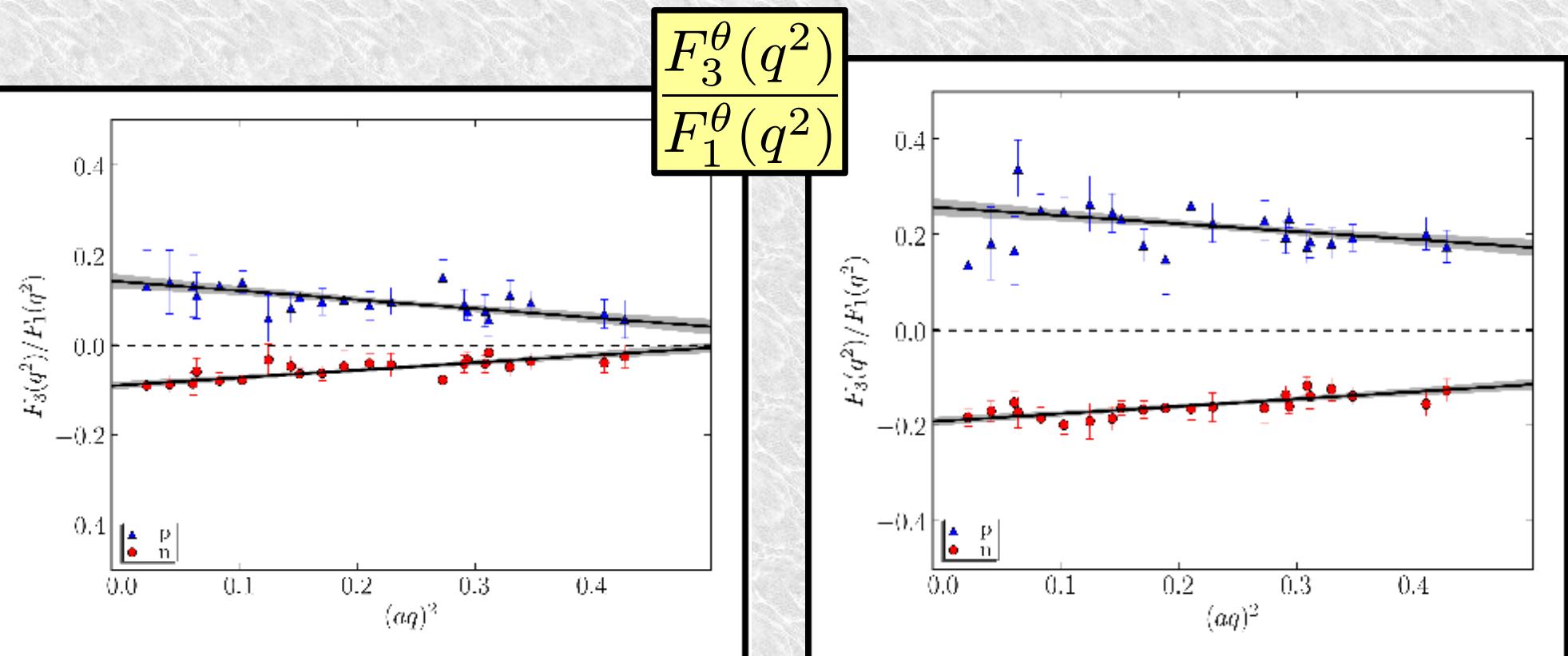
$$D_n = \lim_{q^2 \rightarrow 0} \frac{e}{2m_N} F_3(q^2)$$

$$V_\mu^{EM} = \sum_q e_q \bar{q} \gamma_\mu q$$

Preliminary results of F_3

- valence theta = sea theta = 0.2 (left) and 0.4 (right)
- Dipole ansatz

$$F_3^\theta(q^2) = \frac{F_3^\theta(0)}{(1 + q^2/M^2)^2}$$



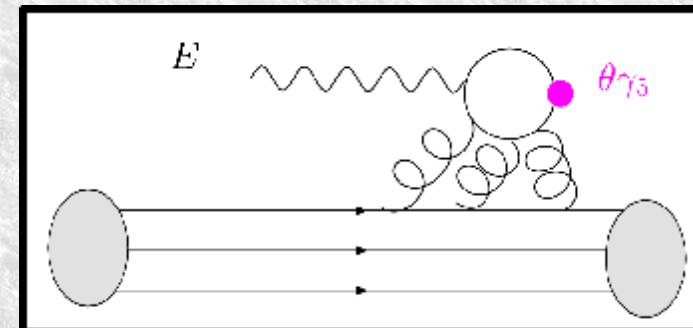
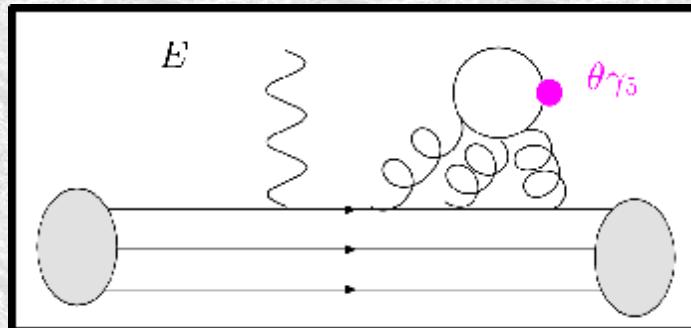
Summary & Discussions

- **QCD+QED DWF simulation** is carried out to determine quark masses: massless quarks are unlikely consistent with experimental spectrum so far(DWF preserves **chiral symmetry**).
- More realistic calculation on **Nf=2+1** is being carried out.
- Dynamical clover simulation with **imaginary θ** term could control **topology distribution** of QCD ensemble.
- Both the **external field method** and the **form factor** calculation show signals for relatively heavy quark mass.
- **(re)normalization of θ** is needed along with the usual mass term, at least, on lattice (for **clover quark, chiral symmetry is broken**).

 More careful studies are needed for physical results.

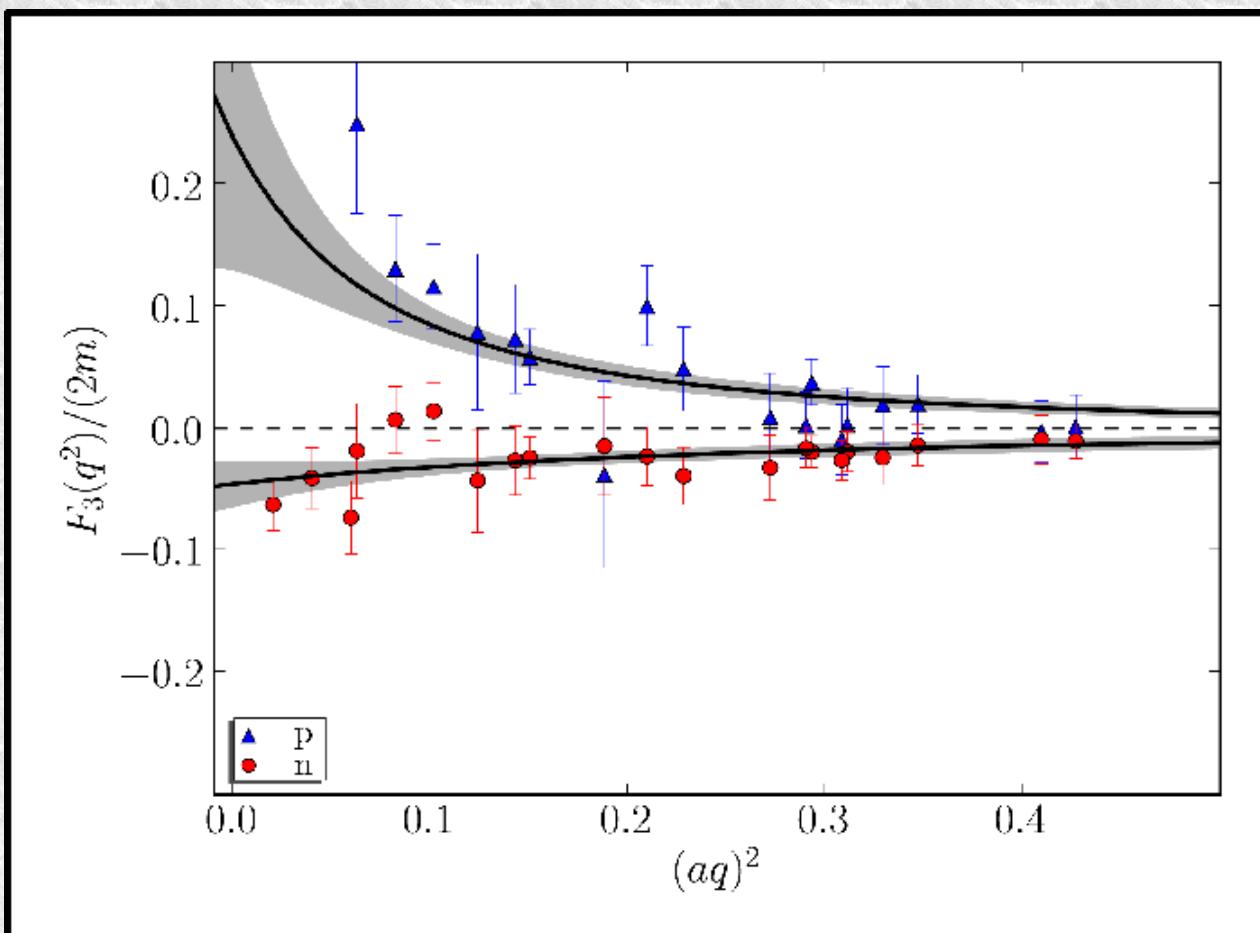
Future plan

- Improving current calculation includes
 - More statistics
 - several θ values to check $\mathcal{O}(\theta^2)$
 - quark mass/ volume/ lattice spacing dependences
 - Valence θ dependence [Aoki-Gocksh-Manohar-Sharpe 90]
- NEDM needs another disconnected quark loop contribution. (07 Shintani et.al.)



Dependence to the valence theta

- [Aoki-Gocksh-Manohar-Sharpe 90] argued valence theta dependence is unphysical lattice artifact and shall be rotated away by U(1) chiral transformation. The clover fermion explicitly breaks chiral symmetry as a lattice artifact (c.f. DWF).
- $\text{theta_val} = 0$, $\text{theta_sea} = 0.4$

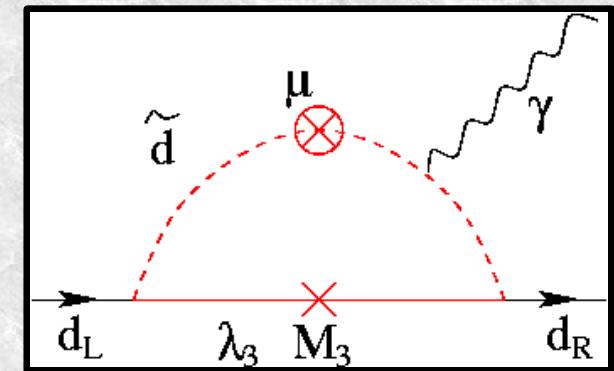


Future prospects

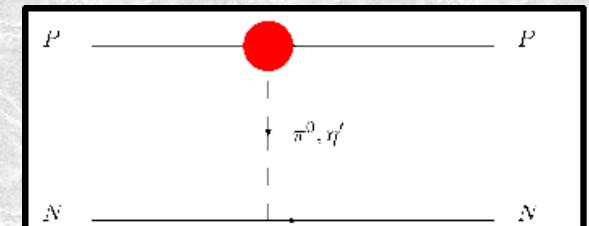
- Effects of Quark EDM on lattice ?

('03 Hisano-Shimizu, Nagai et. al.)

- $i \frac{d_q^C}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 F_{\mu\nu} \psi$ may be doable by $d_q^C \rightarrow -id_q^C$
- Has mixing with other operators



- Study of other Hadrons Deutron, ... , Hg, Tl's EDM ?
measuring CP-violating pi-N-N effective coupling

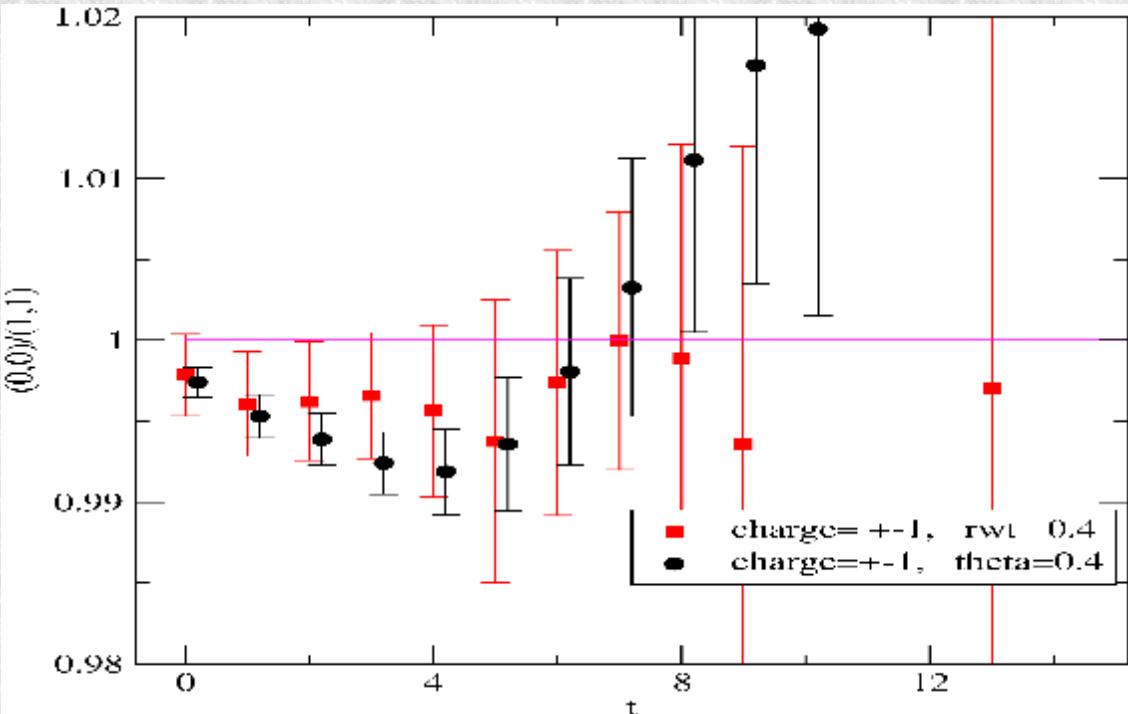


A c k n o w l e d g m e n t s

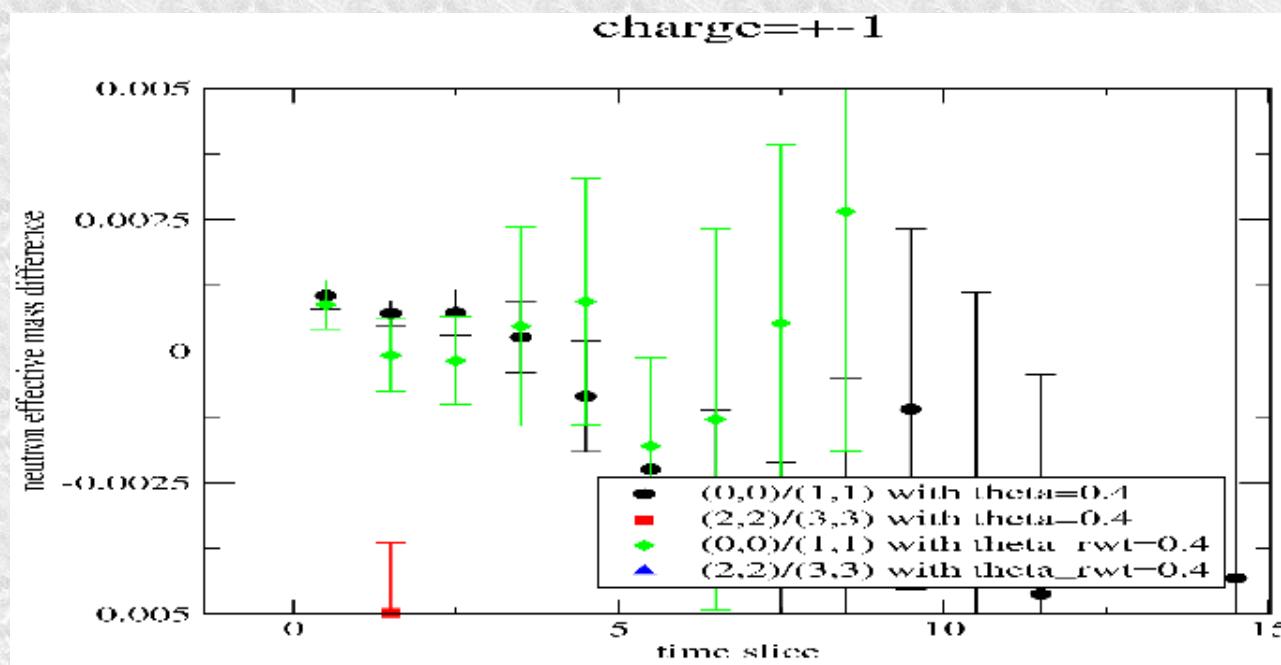
- Nf=2 & 2+1 DWF is generated by the RBC-UKQCD collaboration.
- We are grateful for authors/maintainers of DESY code and CPS++, which are used for ensemble generation and measurements.
- We thank for computational resources supported by the Large Scale Simulation Program No. 07-14 (FY2007) of High Energy Accelerator Research Organization (KEK), the RIKEN Super Combined Cluster (RSCC), and QCDOC at RBRC & Columbia University.
- This work is supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culter, Sports, Science and Technology, No. 17750050 (FY2007-2008), and JSPS & DFG Japan-German Corporative Research Program (FY2008-2009).

Appendix

comparison with reweighting method

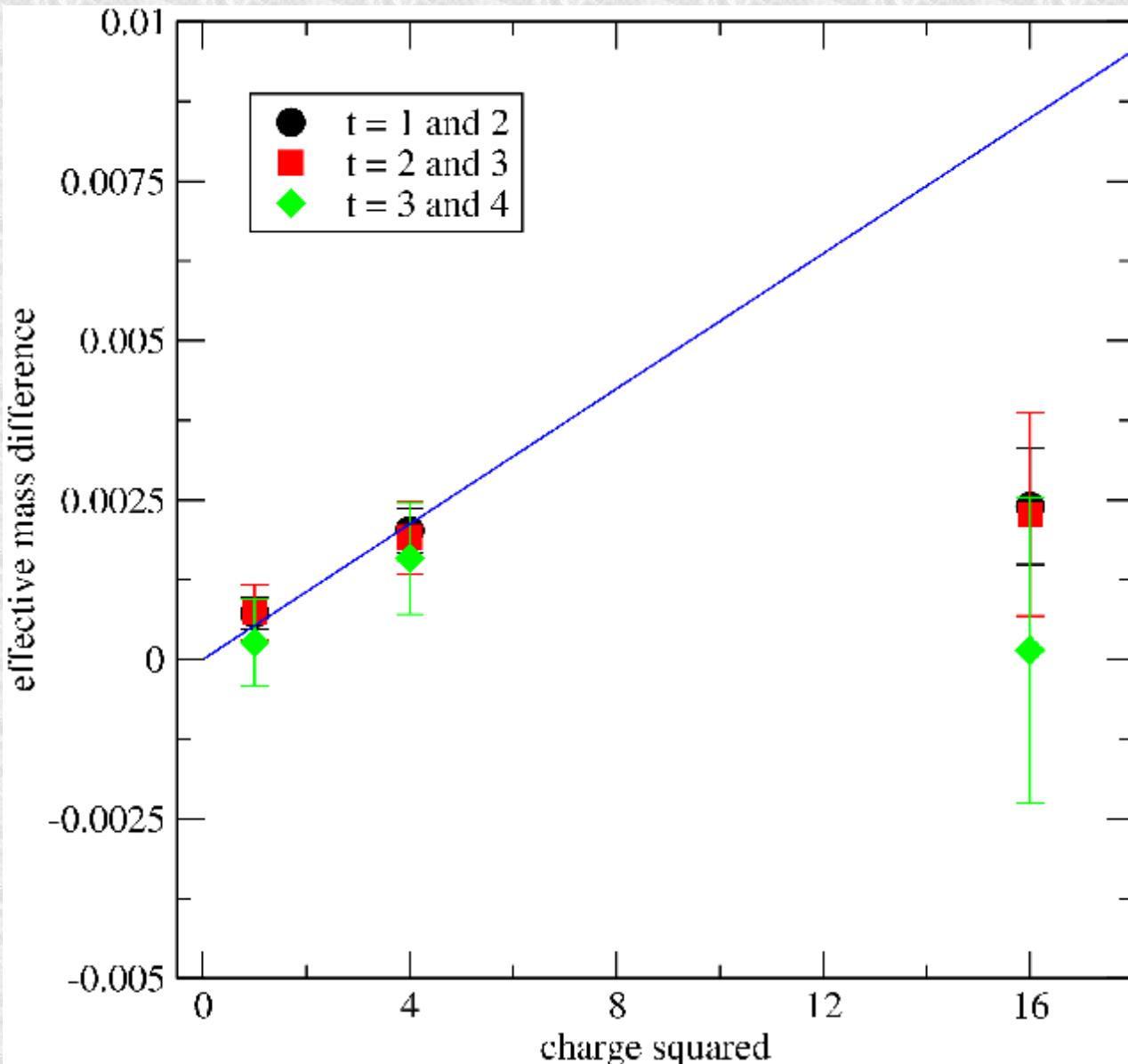


- charge= +/- 1
- reweighting method has larger error bar



- Further nested ratio ?
 - $G(\theta=0)$
 - $G(\text{mag})$

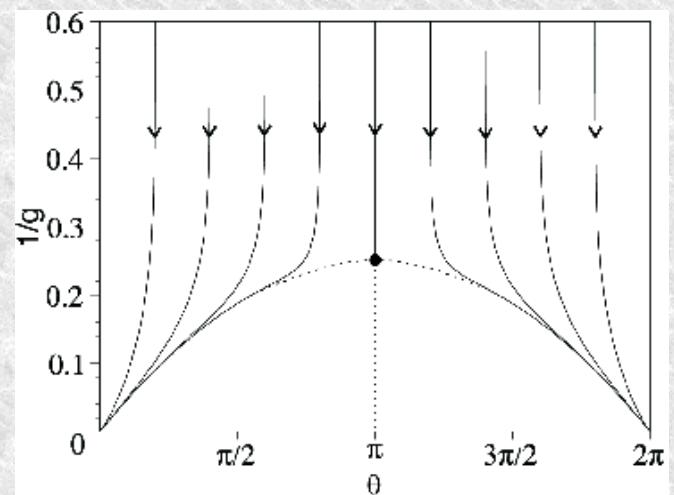
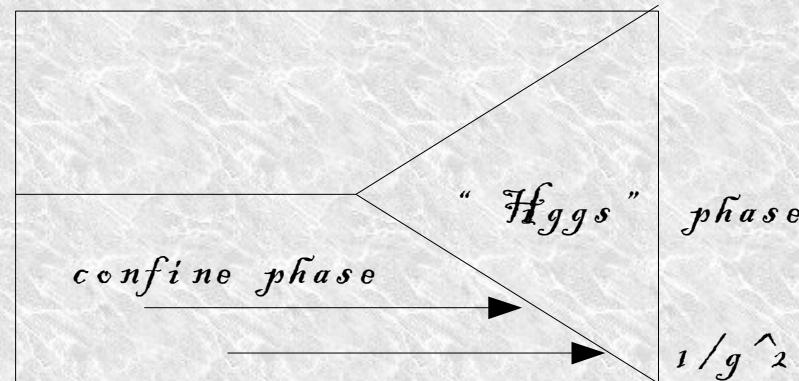
Electric charge dependence



- Consistent with charge squared scaling for smaller two charges.
- Higher order effects is seen for $Q = 4$

renorma liza tion of θ ?

- '94 Schierholz argued an interesting possibility that CP(n) & QCD may **not be confined** in the continuum limit when $\theta \neq 2\pi \times N$
- '06,'07 Apenko : estimations of **RG flow** of theta in Quantum ring.
- '07 Gurosoy, Kiritsis, Nitti, in **Holographic QCD**, suggest the possibility that the **non-perturbatively renormalized theta** runs strongly into zero at IR.



Weak and Uniform E(M) field

- 98 U. Heller, 64 E. Brown construct an EM field on torus which keeps translational invariance,
 $U_{\square,zt} = e^{iE_z t}$ is constant everywhere, equiv. to a “twistier” boundary condition

$$\psi(z, t + N_t) = \exp(iEN_x z)\psi(z, t)$$

The discretization unit is small $E_z^{(\text{unit})} = \frac{2\pi}{N_z N_t} \sim 0.01$

- No **strong negative field at the boundary**, which could skew the energy and/or larger excited state contamination (c.f. **delta function potential** in QM)
- Could also apply to **MDM**, **Polarizability** calcs.

Weak and Uniform external field

- Usual U(1) (unlike Minkowskian used by Aoki-Gocksch, CP-PACS) so that the energy splittings remains real number

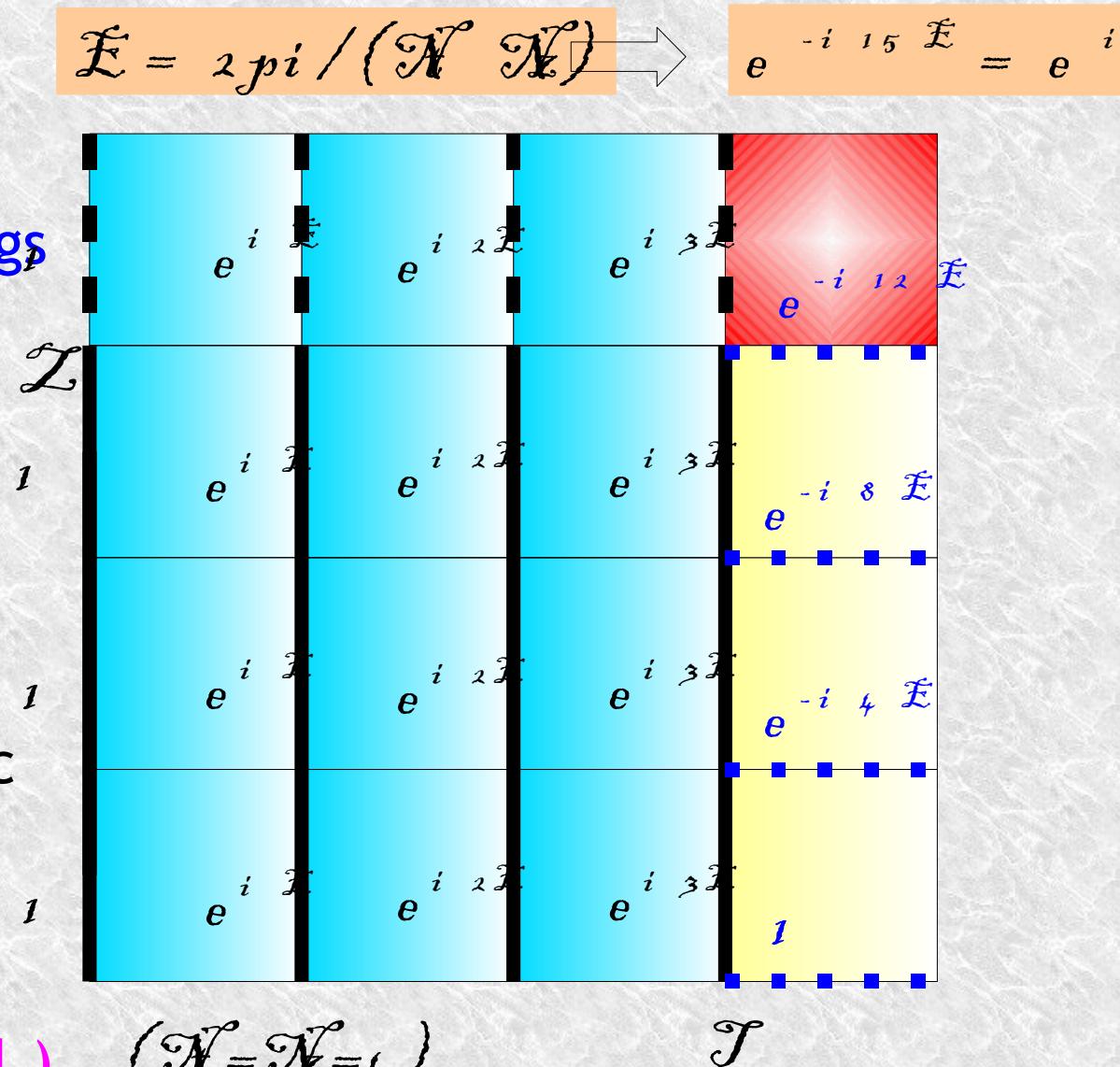
- Weak to avoid E^2 contaminations
- Uniform to avoid a boundary effect, which causes a systematic error due to a large anti-field at the periodic boundary

(05 Shintani et.al.)

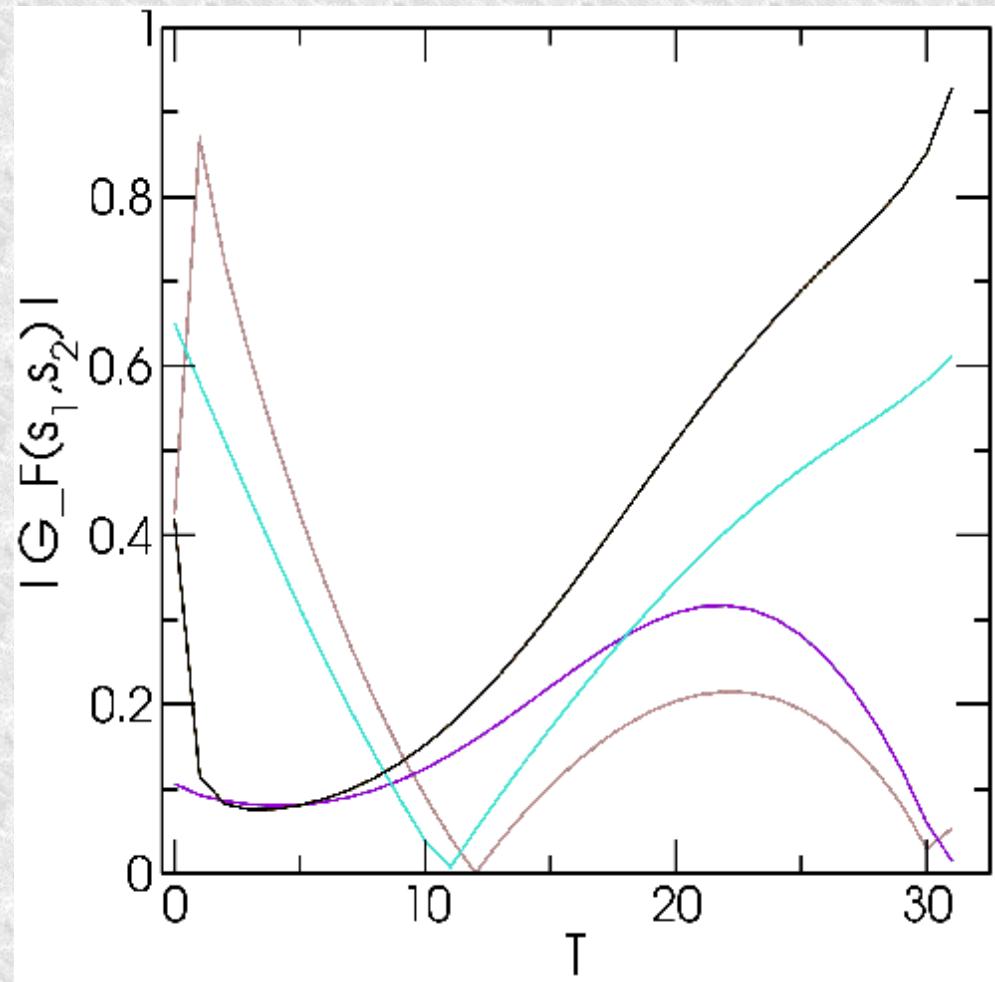
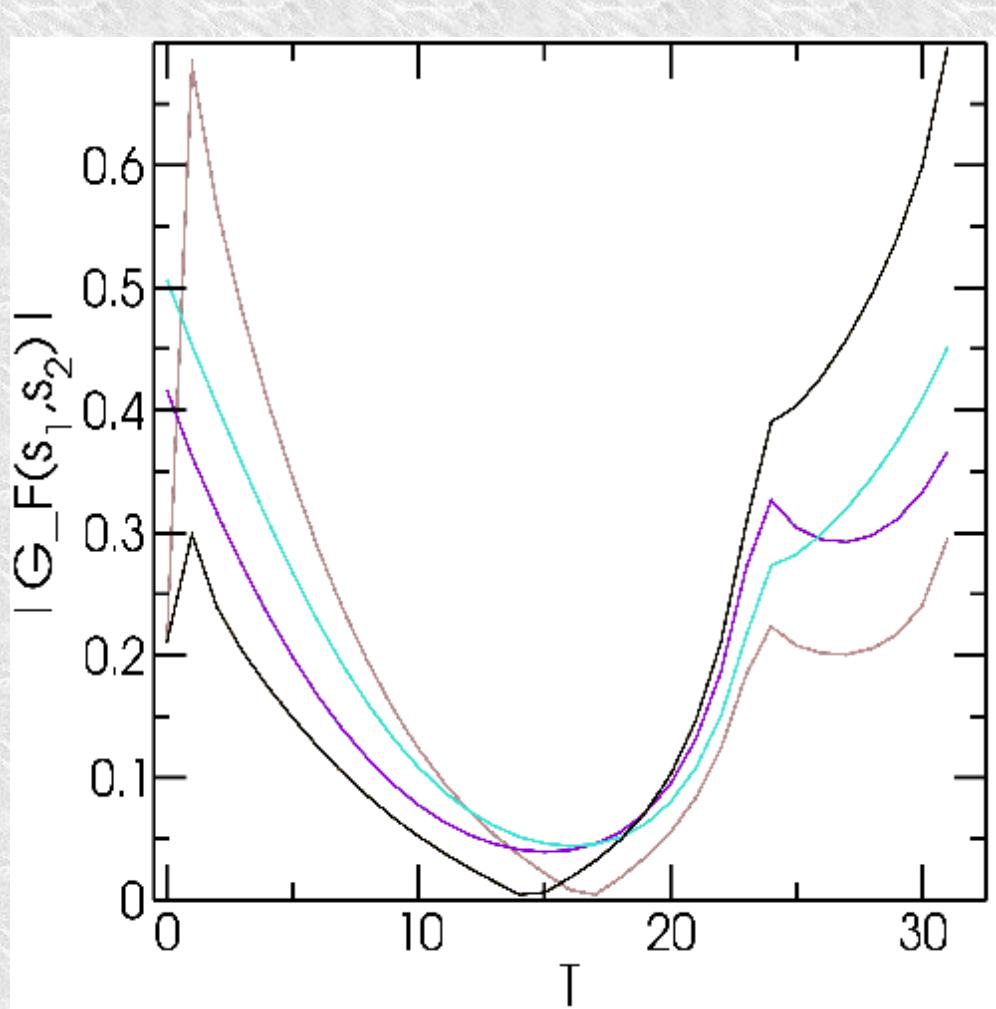
2008

$$(\mathcal{N}=\mathcal{N}=4)$$

Taku Izubuchi



Free case



- $T_{src}=8$

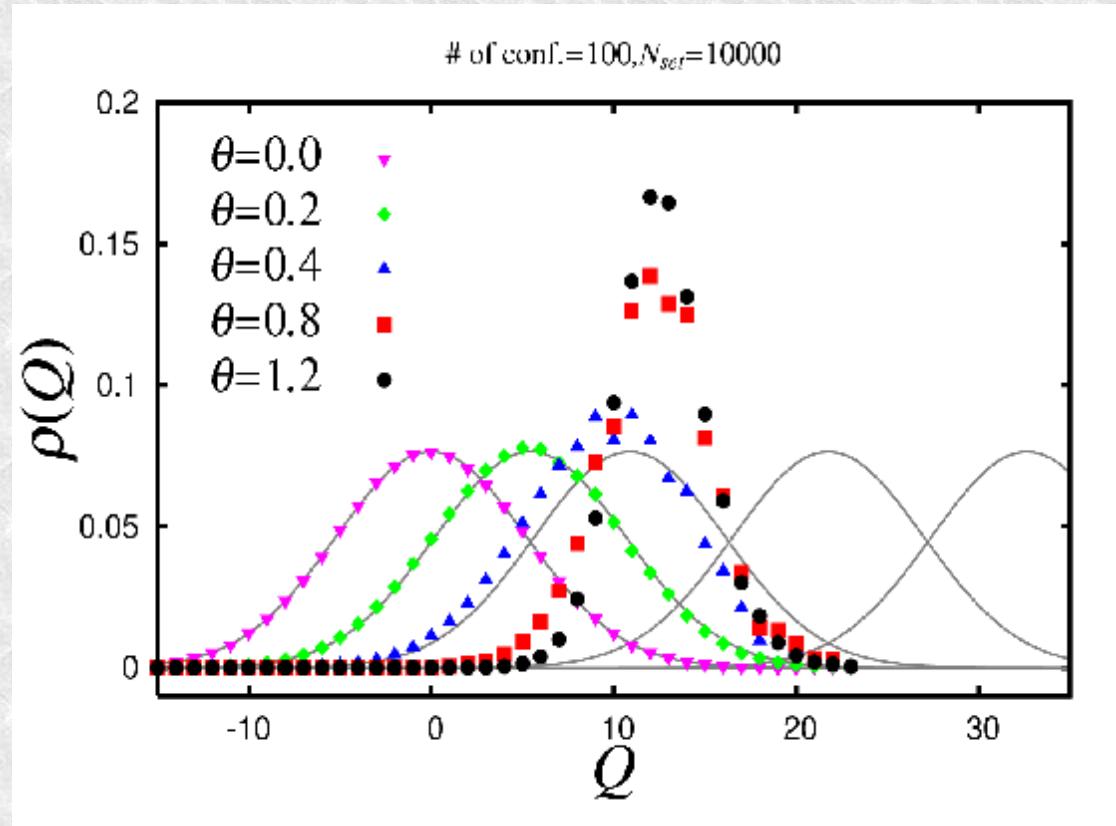
- $T \rightarrow T-8$

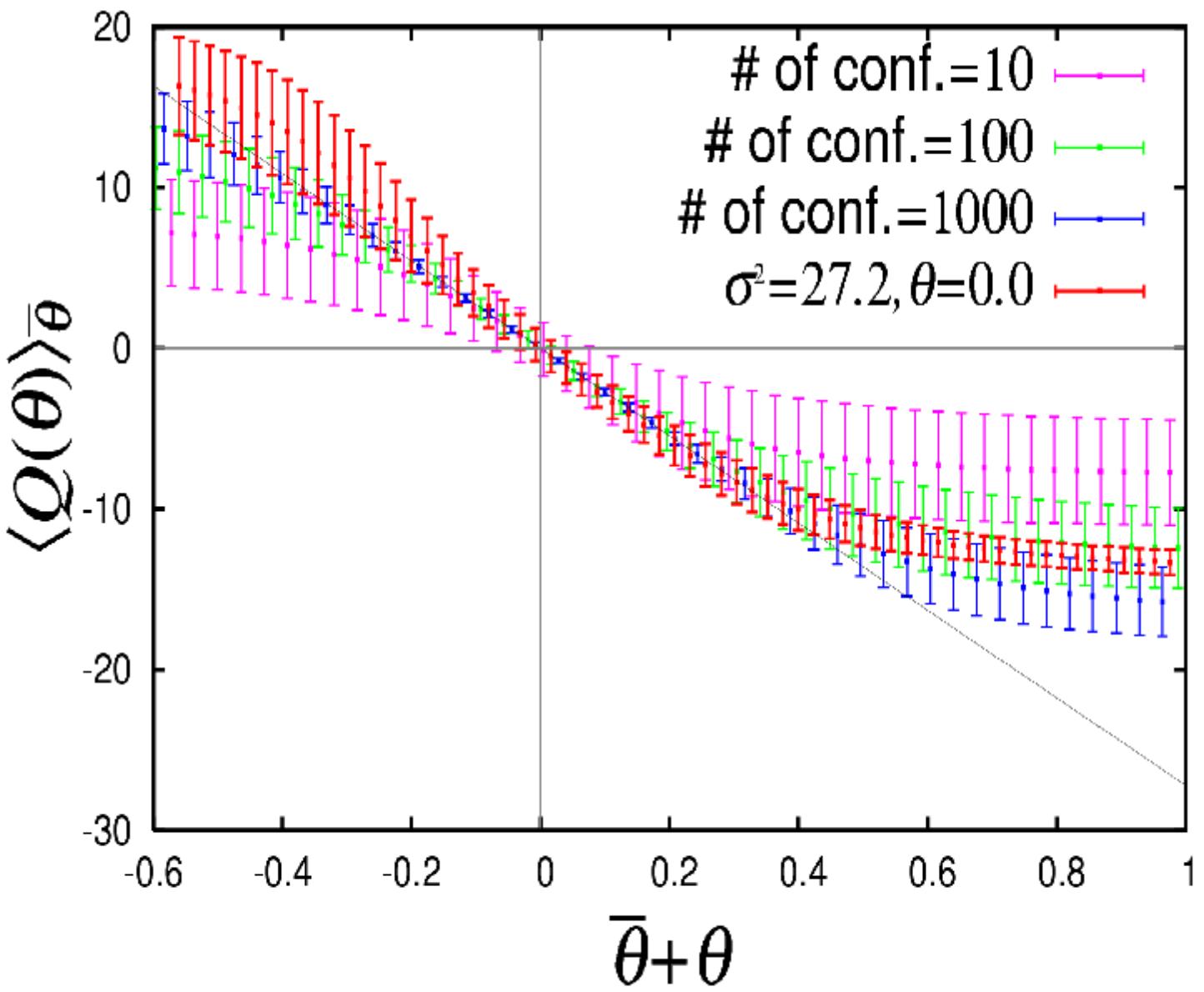
- $E=E(\text{unit}) \times n$

Too much reweighting is dangerous

- An observable $\langle Q e^{\theta Q} \rangle$ *always breaks* the central limit theorem for large enough θ with fixed statistics.
- Gaussian mock-up data

- Nconf=100





- Large enough reweighting gives biased results and underestimated error.
- Compared to theta=0.0 real simulation data result (red), we could get a hint of # of independent configuration, ~ 100 might be appropriate.